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# A Note on Supersymmetric WZW term in Four Dimensions

**Muneto Nitta\***

*Department of Physics, Tokyo Institute of Technology,  
Oh-okayama, Meguro, Tokyo 152-8551, Japan*

## Abstract

We reconsider the supersymmetric Wess-Zumino-Witten (SWZW) term in four dimensions. It has been known that the manifestly supersymmetric form of the SWZW term includes derivative terms on auxiliary fields, the highest components of chiral superfields, and then we cannot eliminate them by their equations of motion. We discuss a possibility for the elimination of such derivative terms by adding total derivative terms. Although the most of derivative terms can be eliminated as in this way, we find that all the derivative terms can be canceled, *if and only if* an anomalous term in SWZW term vanishes. As a byproduct, we find the *first* example of a higher derivative term free from such a problem.

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\* E-mail: nitta@th.phys.titech.ac.jp

When a global symmetry is spontaneously broken down to its subgroup in four dimensions, there appear massless Nambu-Goldstone (NG) bosons. Low-energy effective field theories for these massless bosons are nonlinear sigma models, which can be constructed by the nonlinear realization method [1]. These effective Lagrangian include two derivative terms, and then are dominant at the low-energy scale. Corrections to effective Lagrangian are higher derivative terms. Especially anomalies for the global symmetry can be reproduced at the low-energy scale by the Wess-Zumino-Witten (WZW) term, which is a four derivative term [2, 3].

If spontaneously symmetry breaking occurs in  $N = 1$  supersymmetric field theories in four dimensions, massless NG bosons, with additional bosons and fermionic superpartner, constitute massless chiral superfields [4]. Low-energy effective field theories are described by supersymmetric nonlinear sigma models [5], which can be constructed by a supersymmetric extension of the nonlinear realization method [6]. In these models, anomalies of the global symmetry can be reproduced at the low-energy scale by the supersymmetric extension of WZW (SWZW) term [7, 8, 9]. (For anomalies in supersymmetric gauge theories, see Ref. [10] and references therein.) Manifestly supersymmetric form of the SWZW term has been firstly constructed by Nemeschansky and Rohm [8], in which bosonic parts consist of the bosonic WZW term and an additional four derivative term. Four derivative terms of the Skyrme type also has been considered in Ref. [11]. (See Ref. [12], for higher derivative terms in supersymmetric standard models.) Low-energy effective Lagrangian of supersymmetric QCD including the SWZW term has been discussed in Ref. [13], but only bosonic part has been discussed there.

It was, however, pointed out in Ref. [11] that higher derivative terms including SWZW term have a serious problem: There are terms with spacetime derivatives acting on auxiliary field, namely the highest components of chiral superfields, and then we cannot eliminate them by their equations of motion. Auxiliary fields must propagate! To overcome this problem, Gates and his collaborators has suggested a new form of the SWZW term by doubling the chiral superfields to chiral and complex linear superfields [14]. However this doubling has not been justified yet in the framework of  $N = 1$  supersymmetric low-energy effective field theories, since

there is no Nambu-Goldstone theorem for additional superfields.<sup>1</sup>

In this note we reconsider the SWZW term in supersymmetric nonlinear sigma models consisting of chiral superfields only. There remain a possibility for the elimination of derivative terms on auxiliary fields by adding total derivative terms. We discuss this possibility and find that most derivative terms can be eliminated in this way, but there remain some non-vanishing derivative terms. The condition for disappearance of these terms is equivalent to the condition on the vanishing of an anomalous term. Then this problem cannot be avoided in supersymmetric nonlinear sigma models with the SWZW term, which correctly reproduce anomalies. We find, however, the *first* example of higher derivative terms free from this problem.<sup>2</sup> After we recapitulate the bosonic WZW term and supersymmetric nonlinear sigma models without the WZW term, we discuss the SWZW term.

Before discussing supersymmetric field theories, we recall fundamental facts of the WZW term in bosonic field theories to fix notations. The Lagrangian of nonlinear sigma models is

$$\mathcal{L} = \frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j. \quad (1)$$

where  $\phi^i(x)$  are NG fields for a global symmetry breaking of  $G$  to  $H$ . The number of NG fields is  $i = 1, \dots, \dim(G/H)$ , and  $g_{ij}(\phi)$  is a metric of coset space  $M = G/H$ . The WZW term in bosonic nonlinear sigma models can be written as the integration of a closed 5-form  $\omega$  on  $M$ ,  $d\omega = 0$ , over the five dimensional extension of the spacetime [3]:

$$S_{\text{WZW}} = c \int_5 \omega(\hat{\phi}^i(x, t)), \quad \hat{\phi}^i(x, 0) = \phi^i(x), \quad (2)$$

where  $\hat{\phi}^i(x, t)$  is an extension of fields to the five dimensions, and  $t$  is an extra coordinate. From the Poincare's lemma,  $\omega$  can be written locally by a potential

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<sup>1</sup> In recent works of Gates and his collaborators, the SWZW term are further studied without the doubling [15]. Although the component level analysis is not completed, there is possibility that these models are also free from the problem of the auxiliary field.

<sup>2</sup> In the nonlinear elecro-dynamics, there exists an example of higher derivative terms free from this problem [16].

4-form  $\lambda$  as  $\omega = d\lambda$ . Then, due to the Stokes' theorem, the WZW term becomes

$$S_{\text{WZW}} = c \int_4 \lambda(\phi(x)) = c \int d^4x \varepsilon^{\mu\nu\rho\sigma} \lambda_{ijkl}(\phi) \partial_\mu \phi^i \partial_\nu \phi^j \partial_\rho \phi^k \partial_\sigma \phi^l, \quad (3)$$

which is the integrated WZW term.

In the case of the supersymmetric nonlinear sigma models, NG fields sit in chiral superfields  $\Phi(x, \theta, \bar{\theta})$ , satisfying  $\bar{D}_{\dot{\alpha}} \Phi(x, \theta, \bar{\theta}) = 0$ . A chiral superfield can be expanded by the Grassmannian coordinate  $\theta$  and  $\bar{\theta}$  as [18]

$$\begin{aligned} \Phi(y, \theta) &= A^i(y) + \sqrt{2}\theta\psi^i(y) + \theta\theta F^i(y) \\ \Phi(x, \theta, \bar{\theta}) &= A^i(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A^i(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square A^i(x) + \sqrt{2}\theta\psi^i(x) \\ &\quad - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi^i(x)\sigma^\mu\bar{\theta} + \theta\theta F^i(x), \end{aligned} \quad (4)$$

where  $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ . The Lagrangian of supersymmetric nonlinear sigma models can be written as [5]

$$\int d^4\theta K(\Phi, \Phi^\dagger) = g_{ij^*}(A, A^*)\partial_\mu A^i \partial^\mu A^{*j} + \dots, \quad (5)$$

where dots denote fermionic terms. Using the Kähler potential  $K(\Phi, \Phi^\dagger)$ , the metric is defined by

$$g_{ij^*}(A, A^*) = \frac{\partial^2 K(A, A^*)}{\partial A^i \partial A^{*j}}. \quad (6)$$

Then target manifolds of supersymmetric nonlinear sigma model are Kähler manifolds [5]. Note that, in the calculation of Eq. (5), we must eliminate auxiliary field  $F^i$  of chiral superfields by their equations of motion

$$F^i = \Gamma^i_{jk}(A, A^*)\psi^j\psi^k, \quad (7)$$

where  $\Gamma^i_{jk}$  is the connection on the Kähler manifold.

Let us discuss the supersymmetric analog of the WZW term. The fundamental object for WZW term still be a closed 5-form  $\omega(A, A^*)$  on the Kähler manifold. Situations are, however, slightly different from bosonic theories. The closed condition on  $\omega$  in the Kähler manifold is

$$d\omega = (\partial + \bar{\partial})\omega = 0, \quad (8)$$

where  $\partial$  and  $\bar{\partial}$  are exterior derivatives with respect to holomorphic and anti-holomorphic coordinates  $A^i$  and  $A^{*i}$ , respectively. (For a review of geometries of Kähler manifolds, see, e.g., a textbook [17].) In this case,  $\omega$  can be locally written as

$$\omega = \partial \bar{\partial} \beta, \quad (9)$$

where  $\beta(A, A^*)$  is a potential  $(2, 1)$ -form:  $\beta = \beta_{ijk^*} dA^i \wedge dA^j \wedge dA^{*k}$ .<sup>3</sup> The manifestly supersymmetric expression of SWZW term is given in Ref. [8] by

$$S_{\text{SWZW}} = ic \int d^4x d^4\theta \left[ \beta_{ijk^*}(\Phi, \Phi^\dagger) D^\alpha \Phi^i \sigma^\mu_{\alpha\dot{\beta}} \partial_\mu \Phi^j \bar{D}^{\dot{\beta}} \Phi^{*k} + \text{c.c.} \right]. \quad (10)$$

It was shown in Ref. [8] that if we put  $F^i = 0$  by hand in the bosonic part  $\mathcal{L}_{\text{boson}}$ , it becomes

$$\begin{aligned} \mathcal{L}_{\text{boson}}|_{F^i=0} &= -4i\varepsilon^{\mu\nu\rho\sigma} \lambda_{ijk^*l^*}(A, A^*) \partial_\mu A^i \partial_\nu A^j \partial_\rho A^{*k} \partial_\sigma A^{*l} \\ &\quad - 8\chi_{ijk^*l^*}(A, A^*) \partial_\mu A^i \partial^\mu A^{*k} \partial_\nu A^j \partial^\nu A^{*l}. \end{aligned} \quad (11)$$

Here  $(2, 2)$ -forms  $\lambda$  and  $\chi$  are defined by

$$\lambda_{ijk^*l^*} = \beta_{ijk^*,l^*} - \bar{\beta}_{k^*l^*i,j}, \quad \lambda = \bar{\partial}\beta - \partial\bar{\beta}, \quad (12)$$

$$\chi_{ijk^*l^*} = \beta_{ijk^*,l^*} + \bar{\beta}_{k^*l^*i,j}, \quad \chi = \bar{\partial}\beta + \partial\bar{\beta}, \quad (13)$$

where “,” denote differentiations with respect to scalar fields. The first term proportional to  $\lambda$  in Eq. (11) is just the bosonic part of the (integrated) SWZW term, which is the complex extension of Eq. (3); on the other hand, the second term proportional to  $\chi$  is a non-anomalous  $G$ -invariant term. The 5-form  $\omega$  can be written by these forms as

$$\omega = \partial\lambda = \partial\chi. \quad (14)$$

Although we have put  $F^i = 0$  by hand in the calculation of Eq. (11), we have to eliminate auxiliary field  $F^i$  by their equations of motion for a complete calculation, as done in Eq. (7). There are, however, terms with spacetime derivatives acting on  $F^i$ , and we cannot eliminate them by their equations of motion. There remains a

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<sup>3</sup> Therefore  $\beta$  is a  $(2, 1)$ -form and  $\omega$  is a  $(3, 2)$ -form.

possibility to cancel the derivative terms by adding total derivative terms, which do not change the action, or by partial integrals. In this note we discuss this possibility. Any term of type  $X(A, \psi)\partial_\mu F$ , in which  $X(A, \psi)$  does not include  $F^i$ , can be replaced with  $-F\partial_\mu X(A, \psi)$  by adding a total derivative term  $-\partial_\mu [X(A, \psi)F]$ . Dangerous terms are  $F\partial_\mu F$  terms. (There are no  $\partial_\mu F\partial_\nu F$  terms as shown below.)

Let us calculate  $F\partial_\mu F$  terms for concreteness. The pull-back factors  $\partial_\mu\Phi^i$  and  $D_\alpha\Phi^i$  in Eq. (10) can be written by component fields as

$$\begin{aligned}\partial_\mu\Phi^i &= \partial_\mu A^i + i\theta\sigma^\nu\bar{\theta}\partial_\mu\partial_\nu A^i + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square\partial_\mu A^i + \sqrt{2}\theta\partial_\mu\psi^i \\ &\quad - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\partial_\nu\psi^i\sigma^\nu\bar{\theta} + \theta\theta\underline{\partial_\mu F^i},\end{aligned}\tag{15}$$

$$\begin{aligned}D_\alpha\Phi^i &= \sqrt{2}\psi_\alpha^i + 2\theta_\alpha F^i + 2i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu A^i + \theta_\alpha\bar{\theta}\bar{\theta}\square A^i - \frac{i}{\sqrt{2}}(\theta\sigma_\mu\bar{\theta})(\sigma^\mu\bar{\sigma}^\nu\partial_\nu\psi^i)_\alpha \\ &\quad + i\sqrt{2}(\sigma_\mu\bar{\theta})_\alpha\theta\partial_\mu\psi^i - \frac{1}{2\sqrt{2}}\theta\theta\bar{\theta}\bar{\theta}\square\psi_\alpha^i + i\theta\theta(\sigma^\mu\bar{\theta})_\alpha\underline{\partial_\mu F^i},\end{aligned}\tag{16}$$

where the argument  $x$  of component fields in the right-hand-side are implicit. The last terms associated with underlines in both equations are just dangerous  $\partial_\mu F$  terms.

On the other hand, the tensor part  $\beta_{ijk^*}(\Phi, \Phi^\dagger)$  includes no  $\partial_\mu F$  terms. To expand it by  $\theta$  and  $\bar{\theta}$ , we note the Taylor expansion of  $\beta$  around  $\Phi = 0$ ,

$$\beta_{ijk^*}(\Phi, \Phi^\dagger) = \sum_{N,M=0}^{\infty} \frac{1}{N!M!} \beta_{ijk^*, i_1 \dots i_N j_1^* \dots j_M^*} |_0 \Phi^{i_1} \dots \Phi^{i_N} \Phi^{\dagger j_1} \dots \Phi^{\dagger j_M},\tag{17}$$

where coefficients are evaluated at  $\Phi = 0$ . If we define

$$K_{N,M}(\Phi, \Phi^\dagger) = \Phi^{i_1} \dots \Phi^{i_N} \Phi^{\dagger j_1} \dots \Phi^{\dagger j_M},$$

its expansion by  $\theta$  and  $\bar{\theta}$  can be calculated, to yield

$$\begin{aligned}K_{N,M}(\Phi, \Phi^\dagger) &= K_{N,M}(A, A^*) + \theta(\dots) + \bar{\theta}(\dots) + \theta\theta \left[ F^i \partial_i K_{N,M}(A, A^*) + \dots \right] \\ &\quad + \bar{\theta}\bar{\theta} \left[ F^{*i} \partial_{i^*} K_{N,M}(A, A^*) + \dots \right] + \theta\sigma^\mu\bar{\theta}(\dots) \\ &\quad + \theta\theta\bar{\theta} \left[ \sqrt{2}\bar{\psi}^i F^j \partial_{i^*} \partial_j K_{N,M}(A, A^*) + \dots \right] + \bar{\theta}\bar{\theta}\theta \left[ \sqrt{2}\psi^i F^{*j} \partial_i \partial_{j^*} K_{N,M}(A, A^*) + \dots \right] \\ &\quad + \theta\theta\bar{\theta}\bar{\theta} \left[ F^{*i} F^j \partial_{i^*} \partial_j K_{N,M}(A, A^*) - \frac{1}{2} F^{*i} \psi^j \psi^k \partial_{i^*} \partial_j \partial_k K_{N,M}(A, A^*) \right. \\ &\quad \left. - \frac{1}{2} F^i \bar{\psi}^j \bar{\psi}^k \partial_i \partial_{j^*} \partial_{k^*} K_{N,M}(A, A^*) + \dots \right],\end{aligned}\tag{18}$$

where “. . .” denote terms not including  $F^i$ . (For our purpose, it is sufficient to calculate terms concerning with  $F^i$ .) By summing up the Taylor expansion (17) again, we obtain the expansion of  $\beta$  by  $\theta$  and  $\bar{\theta}$ , given by

$$\begin{aligned} & \beta_{ijk^*}(\Phi, \Phi^\dagger) \\ &= \beta_{ijk^*} + \theta(\cdots) + \bar{\theta}(\cdots) + \theta\theta \left[ F^l \beta_{ijk^*,l} + \cdots \right] + \bar{\theta}\bar{\theta} \left[ F^{*l} \beta_{ijk^*,l^*} + \cdots \right] + \theta\sigma^\mu \bar{\theta}(\cdots) \\ &+ \theta\theta\bar{\theta} \left[ \sqrt{2}\bar{\psi}^m F^l \beta_{ijk^*,lm^*} + \cdots \right] + \bar{\theta}\bar{\theta}\theta \left[ \sqrt{2}\psi^l F^{*m} \beta_{ijk^*,lm^*} + \cdots \right] \\ &+ \theta\theta\bar{\theta}\bar{\theta} \left[ F^{*m} F^l \beta_{ijk^*,lm^*} - \frac{1}{2} F^{*n} \psi^l \psi^m \beta_{ijk^*,lmn^*} - \frac{1}{2} F^l \bar{\psi}^m \bar{\psi}^n \beta_{ijk^*,lm^*n^*} + \cdots \right], \end{aligned} \quad (19)$$

where all  $\beta$  terms at the right-hand-side are evaluated at bosonic fields  $(A, A^*)$ , and “. . .” again denote terms not including  $F^i$ .

By calculating the  $\theta\theta\bar{\theta}\bar{\theta}$  term in Eq. (10) comprising of (19), (16), (15) and the complex conjugation of (16), the  $F\partial_\mu F$  terms can be obtained, to give

$$\begin{aligned} & 2\theta\theta\bar{\theta}\bar{\theta} \left[ \{i(F^i \partial_\mu F^{*k} + F^{*k} \partial_\mu F^i) \partial^\mu A^j \beta_{ijk^*}(A, A^*) + \text{c.c.}\} \right. \\ & \left. - \{F^{*k} \partial_\mu F^j (\bar{\psi}^l \bar{\sigma}^\mu \psi^i) \beta_{ij[k^*,l^*]}(A, A^*) + \text{c.c.}\} \right], \end{aligned} \quad (20)$$

where  $[ , ]$  represent anti-symmetrization of indices. ( $T_{[ij]} = T_{ij} - T_{ji}$ .) Since  $\partial_\mu F$  terms in Eqs. (16) and (15) are proportional to  $\theta\theta$ , there are no  $\partial_\mu F \partial_\nu F$  terms in the  $\theta\theta\bar{\theta}\bar{\theta}$  term in the product. The first two terms in Eq. (20) can be written as  $\partial_\mu(F^{*k} F^i) \partial^\mu A^j \beta_{ijk^*} + \text{c.c.}$ , and then they can be replaced with  $-F^{*k} F^i \partial_\mu(\partial^\mu A^j \beta_{ijk^*}) + \text{c.c.}$  by adding total derivative terms. On the other hand, by adding a total derivative term, the last two terms can be replaced with

$$-2\theta\theta\bar{\theta}\bar{\theta} F^{*k} \partial_\mu F^j (\bar{\psi}^l \bar{\sigma}^\mu \psi^i) [\beta_{ij[k^*,l^*]}(A, A^*) - \bar{\beta}_{k^*l^*[i,j]}(A, A^*)], \quad (21)$$

and this cannot be eliminated by adding any total derivative term. Hence the necessary and sufficient condition for the last term to vanish is

$$\beta_{ij[k^*,l^*]} = \bar{\beta}_{k^*l^*[i,j]}, \quad \bar{\partial}\beta = \partial\bar{\beta}, \quad (22)$$

which gives

$$\lambda = 0, \quad \chi = 2\bar{\partial}\beta = 2\partial\bar{\beta}. \quad (23)$$

We thus have found that the auxiliary fields  $F^i$  can be free from the derivative problem *if and only if* the anomalous term  $\lambda$  disappears. In this condition, however, the non-anomalous higher derivative term of the  $(2, 2)$ -form  $\chi$  in Eq. (11) survives, which is the *first* example of a higher derivative term free from propagating auxiliary fields. From Eq. (23) and  $\partial^2 = \bar{\partial}^2 = 0$ ,  $\chi$  is a closed  $(2, 2)$ -form:

$$d\chi = (\partial + \bar{\partial})\chi = 0. \quad (24)$$

We have discussed the manifestly supersymmetric extension of the WZW term, and have found that the auxiliary field problem cannot be avoided unless an anomalous term vanish. We have found the first example of a higher derivative term in which auxiliary field is free from derivative terms.

This term would play a role in the construction of manifestly supersymmetric soliton, like the Skyrme. A manifestly supersymmetric extension of the original Skyrme term is also an interesting task. A classification of possible higher derivative terms would be needed for supersymmetric chiral perturbation theories. We hope that these studies contribute to developments of supersymmetric field theories.

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